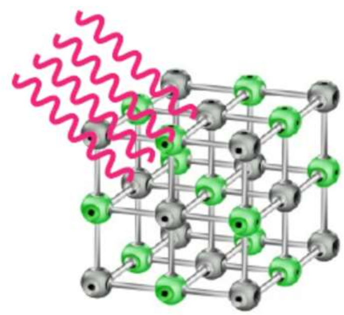
Dynamical Cooper pairing in non-equilibrium electron-phonon systems

Eugene Demler Harvard University

Collaborators:
Mehrtash Babadi (Caltech/Broad), Michael Knap (TU Munich),
Ivar Martin (Argonne), Gil Refael (Caltech)



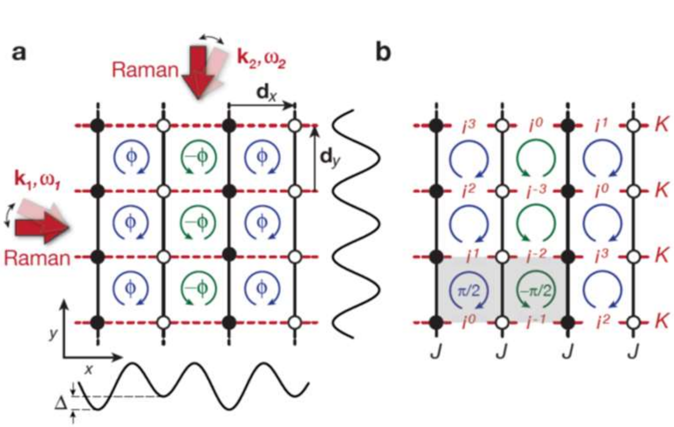
Harvard-MIT CUA

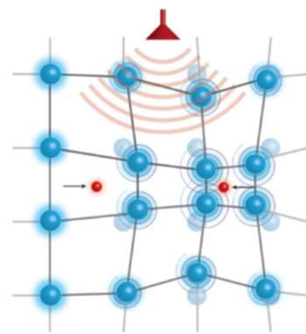
$$ NSF DMR, Simons Foundation,
MURI QUANTUM MATTER, ITS ETH

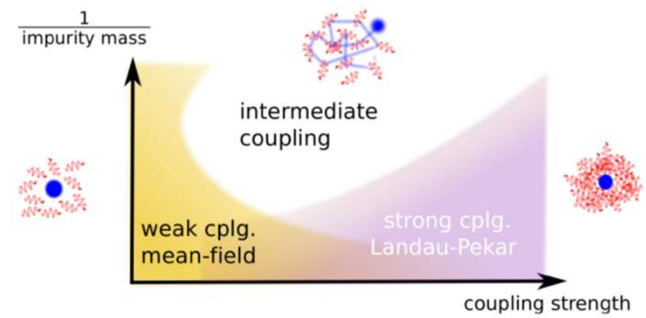
Ultracold atoms and photoinduced superconducitivty

Gauge potential engineering in cold atoms, e.g. Bloch group 2012

Bose polarons







Also expts by Sengstock, Arimondo, Oberthaler, Bloch, Chin, Spielman, Hemmerich, Esslinger, Ketterle, Greiner, Zwierlein

Experiments by Sengstock, Esslinger, Arlt, Jin, Pfau, Oberthaler, Bloch, Killian, …

New tool: Floquet engineering of interactions

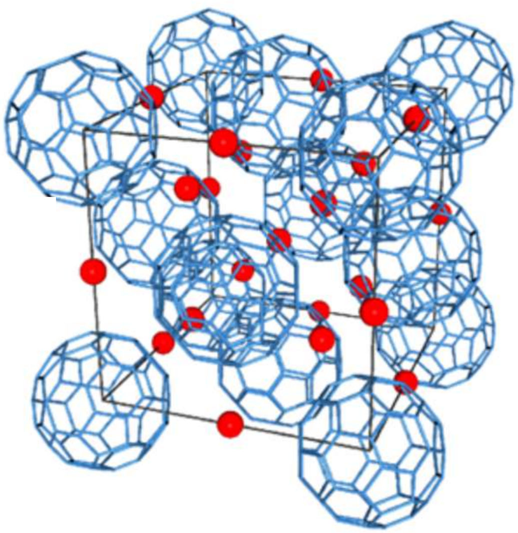
Introduction:
experimental evidence of photo-enhanced superconductivity in electron-phonon superconductor K_3C_{60}

Other experiments: optical control of Mott insulators,
charge and spin density wave states,
superconductivity in high Tc cuprates

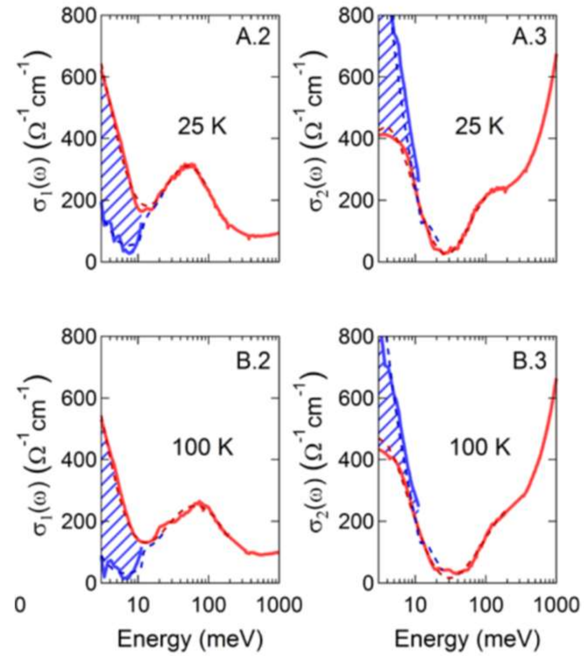
Motivation: photoinduced superconducitivty in K3C60

M. Mitrano et al.,
Nature 530, 461 (2016)

Tc=20K



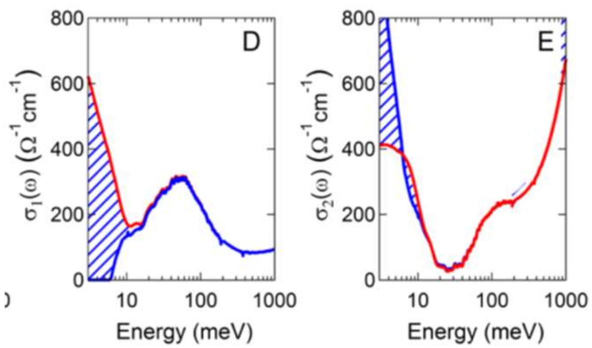
Response of photoexcited system



T=25 K

T=100 K

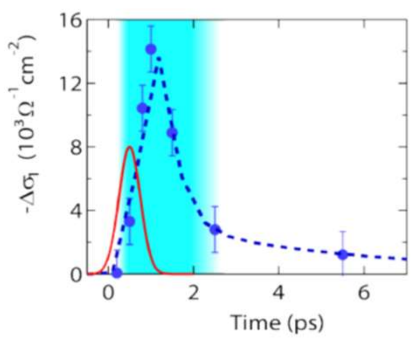
Equilibrium: T=25K (red) and T= 10 K (blue)



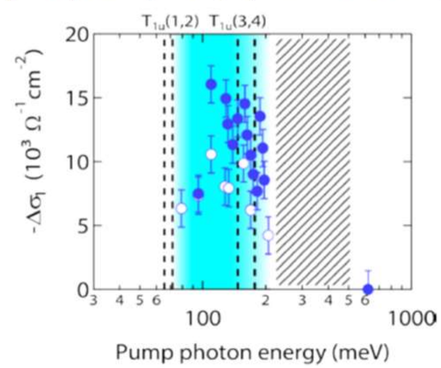
Motivation

Light-induced superconductivity in K_3C_{60}

Dynamics of the induced optical gap



Induced spectral weight loss vs. pump frequency (const. fluence)



Resonance with phonons makes this effect very different from Wyatt-Dayem effect

Outline

Physical picture: enhanced electron-phonon interaction in systems with driven phonons

Floquet-Migdal-Eliashberg analysis

Simpler analysis: polaron transformation

Conclusions

How to increase electron-phonon interaction
and enhance superconductivity, CDW, …

This talk: focus on superconductivity

Electron-phonon interactions in systems with driven phonons

$$\mathcal{H} = \sum_q \left(\frac{P_q^2}{2M} + \frac{M\omega_q^2}{2} Q_q^2 \right) + \mathcal{H}_{\mathrm{drive}}(t)$$

Example: parametric drive of phonons

$$\mathcal{H} = \sum_q \left(\frac{P_q^2}{2M} + \frac{M\Omega_q^2(t)}{2} Q_q^2 \right)$$

$$\Omega_q^2(t) = \omega_q^2 \left(1 + 2\,\alpha \cos(2\,\Omega_{\mathrm{drv}} t) \right)$$

Effective electron-electron interaction assuming slow electron dynamics

$$\mathcal{H}_{\mathrm{eff}} = U(t)\, \hat{\rho}_{\mathrm{el}}(t)\, \hat{\rho}_{\mathrm{el}}(t)$$

$$U(t) = \frac{|\tilde{g}_{\mathbf{q}}|^2}{\hbar} \int_{-\infty}^{t} \mathrm{d}t' \; \mathcal{D}_{QQ}^{R}(t,t')$$

$$\mathcal{D}_{QQ}^{R}(t,t') = -i\theta(t-t')\langle \hat{Q}(t)\,\hat{Q}(t') - \hat{Q}(t')\,\hat{Q}(t)\rangle$$

Electron-phonon interactions in systems with driven phonons

Compare to

$$\mathcal{H}' = \mathcal{H} - \phi\,\hat{Q}$$

$$\langle\,\hat{Q}\,\rangle = \chi\,\phi$$ Fluctuation-dissipation theorem $$\chi = \langle\,\hat{Q}\,\hat{Q}\,\rangle$$

$$\Delta E = -\frac{\chi\phi^2}{2}$$

From electron-phonon coupling to effective electron-electron interaction

$$\mathcal{H}_{\mathrm{el-phon}} = \sum_{k\,q\,\sigma} g_q Q_q\, c^\dagger_{k-q\sigma}\, c_{k\sigma}$$

$$\mathcal{H}_{\mathrm{eff}} = U(t)\,\hat{\rho}_{\mathrm{el}}(t)\,\hat{\rho}_{\mathrm{el}}(t)$$

$$U(t) = \frac{|\tilde{g}_{\mathbf{q}}|^2}{\hbar}\int_{-\infty}^{t} \mathrm{d}t'\ \mathcal{D}^R_{QQ}(t,t')$$

$$\mathcal{D}^R_{QQ}(t,t') = -i\theta(t-t')\langle\hat{Q}(t)\,\hat{Q}(t') - \hat{Q}(t')\,\hat{Q}(t)\rangle$$

Phonon response function

Parametric drive

$$\mathcal{H} = \sum_q \left(\frac{P_q^2}{2M} + \frac{M\Omega_q^2(t)}{2} Q_q^2 \right)$$

Harmonic oscillator equations of motion

$$\frac{\mathrm{d}\hat{Q}(t)}{\mathrm{d}t} = \frac{\hat{P}(t)}{M}$$

$$\frac{\mathrm{d}\hat{P}(t)}{\mathrm{d}t} = -M\omega_{\mathbf{q}}^2\left[1 + 2\alpha\cos(2\Omega_{\mathrm{drv}}t)\right]\hat{Q}(t)$$

From linearity of equations

$$\hat{Q}(t) = \mathfrak{M}_{QQ}(t,t')\,\hat{Q}(t') - \mathfrak{M}_{QP}(t,t')\,\frac{\hat{P}(t')}{M\Omega_{\mathrm{drv}}}$$

Response function

$$\begin{aligned}\mathcal{D}^R_{QQ}(t,t') &= -i\,\theta(t-t')\langle[\hat{Q}(t'),\hat{P}(t')]\rangle \times \frac{-\mathfrak{M}_{QP}(t,t')}{M\Omega_{\mathrm{drv}}}\\ &= -\frac{\hbar}{M\Omega_{\mathrm{drv}}}\,\theta(t-t')\,\mathfrak{M}_{QP}(t,t')\end{aligned}$$

Response function does not depend on the initial state of phonons.
It is determined by the Hamiltonian only. It is enhanced near parametric resonance.

Electron-phonon interactions in systems with driven phonons

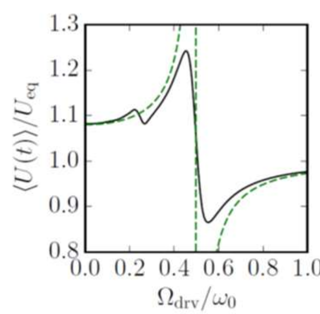
Parametrically driven phonons $\Omega_q^2(t) = \omega_q^2\left(1 + 2\alpha\cos(2\Omega_{\rm drv}t)\right)$

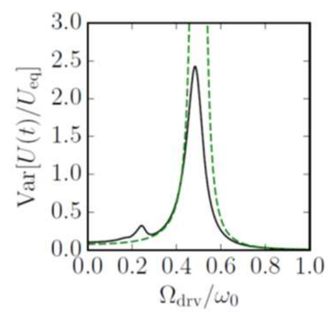
$$\frac{U(t)}{U_{\rm eq}} = 1 - \frac{2\alpha\,\omega_{\mathbf{q}}^2\,\cos(2\Omega_{\rm drv}t)}{\omega_{\mathbf{q}}^2 - 4\Omega_{\rm drv}^2} + \frac{2\alpha^2\,\omega_{\mathbf{q}}^2\left[\omega_{\mathbf{q}}^2 - 16\Omega_{\rm drv}^2 + \omega_{\mathbf{q}}^2\cos(4\Omega_{\rm drv}t)\right]}{(\omega_{\mathbf{q}}^2 - 16\Omega_{\rm drv}^2)(\omega_{\mathbf{q}}^2 - 4\Omega_{\rm drv}^2)} + \mathcal{O}(\alpha^4)$$

Effective interaction: time average and variance. $\alpha = 0.2$

$$\left\langle\frac{U(t)}{U_{\rm eq}}\right\rangle = 1 + \frac{2\alpha^2\,\omega_{\mathbf{q}}^2}{\omega_{\mathbf{q}}^2 - 4\Omega_{\rm drv}^2} + \mathcal{O}(\alpha^4)$$

$$\mathrm{Var}\left[\frac{U(t)}{U_{\rm eq}}\right] = \frac{2\,\alpha^2\,\omega_{\mathbf{q}}^4}{(\omega_{\mathbf{q}}^2 - 4\Omega_{\rm drv}^2)^2} + \mathcal{O}(\alpha^4)$$





Strong enhancement near resonance. Large response function near "instability"

"Naïve" averaging

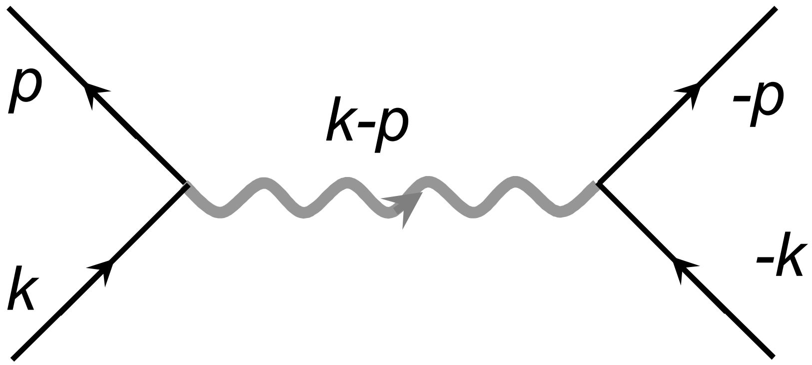
$$\left\langle\frac{T_c^{\rm BCS}(t)}{T_{c,\rm eq}^{\rm BCS}}\right\rangle = \left\langle e^{-\frac{1}{\nu(0)U_{\rm eq}} + \frac{1}{\nu(0)U(t)}}\right\rangle$$

Exponential dependence of Tc: gain on increase in U is larger than suppression due to decrease

Simple argument II

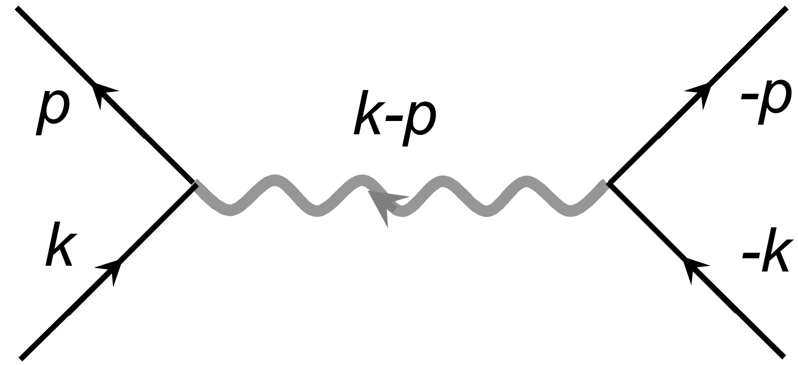
How to increase electron-phonon interaction
and enhance superconductivity, CDW, …

Can one gain from non-equilibrium state of phonons?



interaction via phonon emission

$$-\frac{g^2\,(1+n)}{\epsilon_p+\omega_{k-p}-\epsilon_k}\approx-\frac{g^2\,(1+n)}{\omega_{\rm ph}}$$



interaction via phonon absorption

$$-\frac{g^2\,n}{\epsilon_p-\omega_{p-k}-\epsilon_k}\approx+\frac{g^2\,n}{\omega_{\rm ph}}$$

Effective interaction

$$\boxed{V_{\rm eff}=-\frac{g^2}{\omega_{\rm ph}}}$$

This is the usual argument that real photons do not help to increase effective pairing strength

Driven phonon system

$$\mathcal{H} = \frac{P^2}{2} + \frac{Q^2}{2} + A\cos 2\omega_{\rm dr} t \times Q^2$$

Exact solution is available.

Consider a simplified version that has essential physics

$$\mathcal{H} = \omega_{\rm ph} b^\dagger b + (A e^{-2i\omega_{\rm dr} t} b^\dagger b^\dagger + \text{h.c.})$$

Transform to the rotating frame

$$\tilde{\mathcal{H}} = (\omega_{\rm ph} - \omega_{\rm dr}) b^\dagger b + (A b^\dagger b^\dagger + \text{c.c.})$$

This looks like a Bogoliubov Hamiltonian

Electron-phonon interaction in a driven phonon system

$$\tilde{\mathcal{H}}_{\mathrm{phon\,dr}} = \tilde{\omega}_{\mathrm{ph}}\, b^\dagger b + (\, A\, b^\dagger b^\dagger + \mathrm{c.c.}\,)$$

Bogoliubov transformation diagonalizes the Hamiltonian of driven phonons

$$e^S\, \tilde{\mathcal{H}}_{\mathrm{phon\,dr}}\, e^{-S} = \tilde{\omega}_{\mathrm{ph}}\, b^\dagger b$$

$$\boxed{e^S\, b\, e^{-S} = \cosh\xi\, b + \sinh\xi\, b^\dagger}$$

Bogoliubov transformation amplifies electron-phonon interaction.

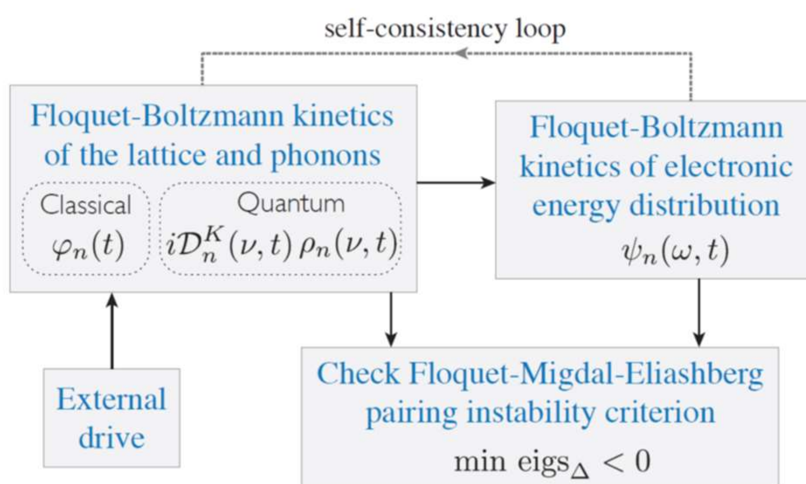
$$\boxed{\mathcal{H}_{\mathrm{el-phon}} = g \sum c^\dagger_{k+q}\, c_k\, (\, b_q + b^\dagger_{-q}\,)}$$

Analogous effect has been pointed out in opto-mechanics: Lemonde et al., arXiv:1509.09238 (2015)

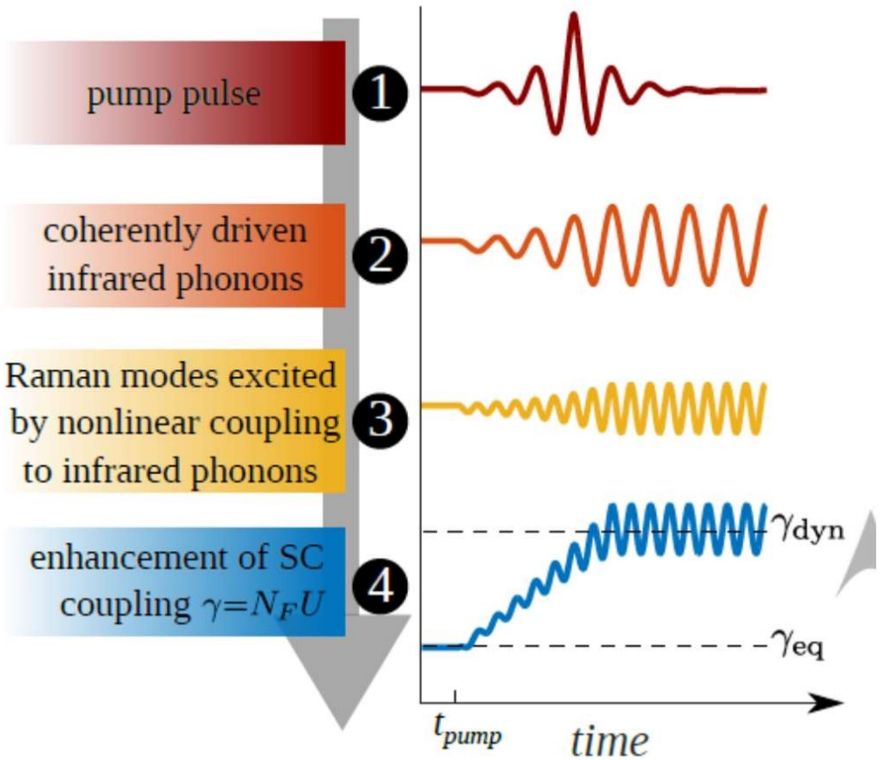
Photo-induced superconductivity

Floquet-Keldysh-Migdal-Eliashberg approach

M. Babadi, M. Knap, G. Refael, I. Martin, E. Demler



Driven electron-phonon system



$$\tau_{\mathrm{decoherence}} \gg \tau_{\mathrm{pairing}}, \tau_{\mathrm{therm}}$$

This talk: focus on dynamical effects

Microscopic model

$$\mathcal{L}[\varphi,\Psi](t) = \sum_{\mathbf{k}} \Psi^\dagger_{\mathbf{k}} \left(i\partial_t \mathbb{I} - \xi_{\mathbf{k}}\hat{\sigma}_3\right)\Psi_{\mathbf{k}} - \frac{1}{2}\sum_{\mathbf{q}} \frac{1}{2\omega_{\mathbf{q}}}\varphi_{\mathbf{q}}\left(\partial_t^2 + \omega_{\mathbf{q}}^2\right)\varphi_{-\mathbf{q}}$$

$$- \sum_{j\in \text{lattice}} \mathcal{V}^{\text{ph}}(\varphi_j) - \frac{1}{\sqrt{N}}\sum_{\mathbf{k},\mathbf{k}'} g_{\mathbf{k},\mathbf{k}'}\,\varphi_{\mathbf{k}-\mathbf{k}'}\,\Psi^\dagger_{\mathbf{k}'}\hat{\sigma}_3\Psi_{\mathbf{k}} + \frac{\Lambda}{2}|F(t)|^2 \sum_{j\in\text{lattice}} \varphi_j$$

Phonons are driven only at q=0: e.g. $\Lambda\varphi^2_{\mathrm{IR},q=0}(t)\varphi_{q=0}$.

This analysis can be applied to the IR modes themselves, if they couple to electrons

We assume phonon-nonlinearities given by κ_3 and κ_4.

$$\mathcal{V}^{\text{ph}}(\varphi) = -\frac{\kappa_3}{3!}\varphi^3 - \frac{\kappa_4}{4!}\varphi^4$$

We assume finite small dissipation γ_0 for phonons at q=0 due to other modes.

Additional dissipation is generated by electrons

Phonon dynamics

Equations of motion for phonons: coherent part and fluctuations

$$\boxed{\frac{1}{2\omega_0}\left(\partial_t^2+\omega_0^2+\gamma_0\partial_t\right)\varphi(t)-\frac{\kappa_4}{6}\,\varphi^3(t)-\frac{\kappa_3}{2}\,\varphi^2(t)\;-\frac{\kappa_4}{2}\,\chi(t)\,\varphi(t)=\frac{\Lambda}{2}\,|F(t)|^2+\frac{\kappa_3}{2}\,\chi(t)}$$

$$\chi(t)\equiv\frac{1}{N}\sum_{\mathbf{q}}i\mathcal{D}_{\mathbf{q}}(t,t)$$

Force from
finite q phonons

$$\boxed{-\frac{1}{2\omega_{\mathbf{q}}}\left[\partial_{t_1}^2\;+\omega_{\mathbf{q}}^2\right]\mathcal{D}_{\mathbf{q}}(t_1,t_2)=\delta_{\mathcal{C}}(t_1,t_2)\;+V(t_1)\,\mathcal{D}_{\mathbf{q}}(t_1,t_2)+\int_{\mathcal{C}}\mathrm{d}\tau\,\Pi_{\mathbf{q}}(t_1,\tau)\,\mathcal{D}_{\mathbf{q}}(\tau,t_2)}$$

$$V(t)\equiv-\frac{\kappa_4}{2}\,\chi(t)-\frac{\kappa_4}{2}\,\varphi^2(t)-\kappa_3\,\varphi(t)$$

Drive from
q=0 phonon

$$\Pi_{\mathbf{q}}(t_1,t_2)=\frac{1}{N}\sum_{\mathbf{k}}|g_{\mathbf{k},\mathbf{k}+\mathbf{q}}|^2\;\mathrm{tr}\left[\hat{\mathcal{G}}_{\mathbf{k}+\mathbf{q}}(t_1,t_2)\,\hat{\sigma}_3\,\hat{\mathcal{G}}_{\mathbf{k}}(t_2,t_1)\,\hat{\sigma}_3\right]$$

Force from electrons
including damping
and frequency renormalization

Floquet-Wigner representation

Wigner transform—

$$\mathcal{D}(t_1,t_2) \to \mathcal{D}(\omega,T) \equiv \int \mathrm{d}t\, e^{-i\omega t}\, \mathcal{D}(T+t/2, T-t/2)$$

center-of-mass (COM) time

$$T = (t_1+t_2)/2$$

Usually:
Slow COM time evolution
Fast relative time.
Not applicable here b/c
of the fast drive

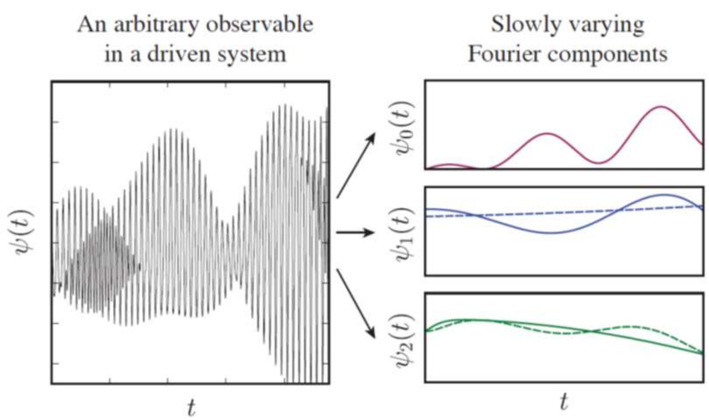
$$\varphi_{\mathrm{IR}}^2(t) = \mathcal{A}(t)\,\cos^2(\Omega t)$$

$$\partial_t \mathcal{A}(t)/\mathcal{A}(t) \ll \Omega^{-1}, \bar{\Omega}_0^{-1}$$

"Floquet-Wigner" representation

$$\mathcal{D}^{R/A/K}(\omega,T) = \sum_n \mathcal{D}_n^{R/A/K}(\omega;T)\, e^{-in\Omega T}$$

slowly-varying
Floquet components



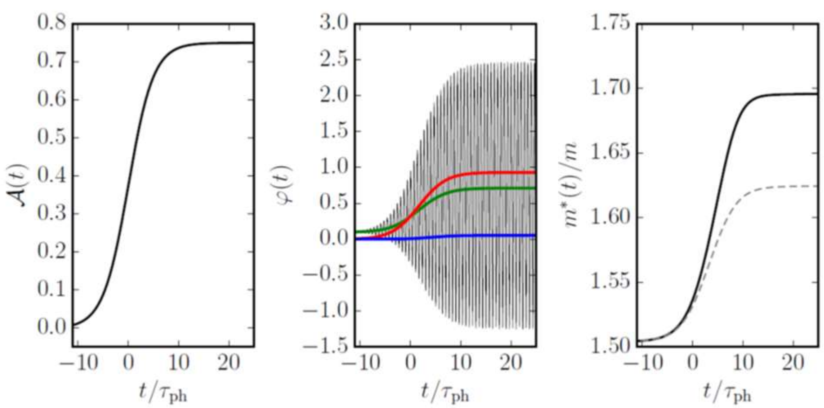
Earlier work on Floquet-Wigner:
Tsuji et al., PRB (2008),
Genske, Rosch, PRA (2015).

This work: full quantum kinetic equation
beyond quasiparticle Boltzmann approximation

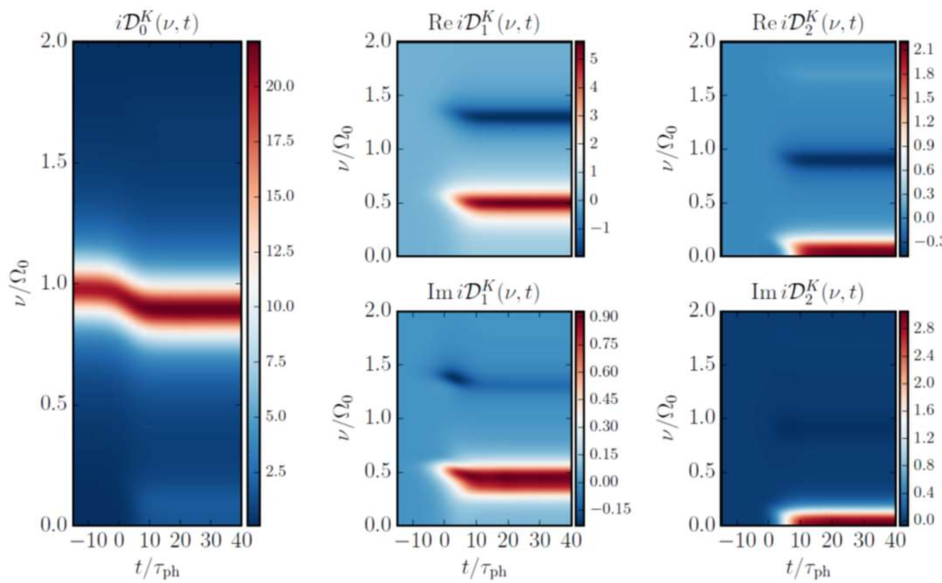
Evolution of phonons

Ramped-up external drive from with Amax=0.75

Coherent amplitude at q=0



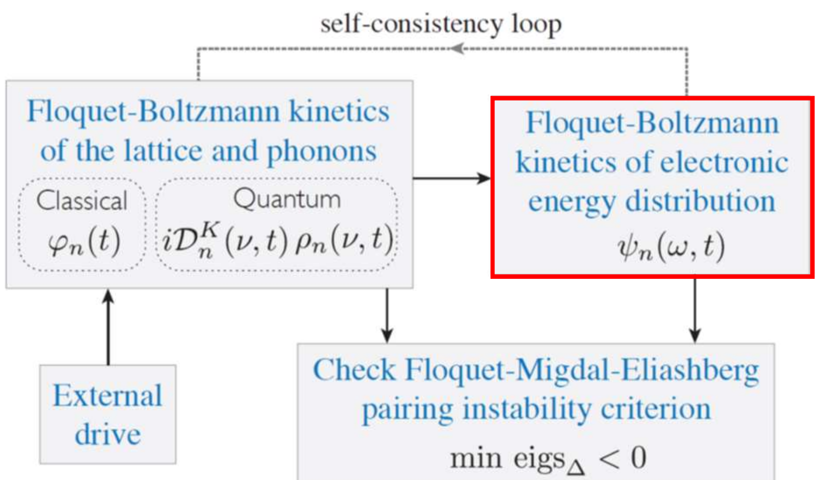
Propagators for finite q



Note the red shift in the spectrum of phonons: contributes to SC enhancement

The real time Migdal-Eliashberg theory

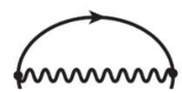
We want to compare enhancement of electron-phonon interaction with shortening of electron lifetime



The real time Migdal-Eliashberg theory

No superconductivity yet

$$\hat{\Sigma}_{\mathbf{k}}(t,t') = \frac{i}{N}\sum_{\mathbf{k}'} \hat{\sigma}_3\, \hat{\mathcal{G}}_{\mathbf{k}'}(t,t')\, \hat{\sigma}_3\, |g_{\mathbf{k}\mathbf{k}'}|^2\, D_{\mathbf{k}-\mathbf{k}'}(t,t')$$



Renormalization of electron energy, quasiparticle weight, lifetime

spectral/Keldysh decomposition

$$i\hat{\mathcal{G}}^{>}_{\mathbf{k}}(\omega,T) = \frac{1}{2}\left[i\hat{\mathcal{G}}^{K}(\omega,T) + \hat{\mathsf{A}}(\omega,T)\right]$$

$$i\hat{\mathcal{G}}^{<}_{\mathbf{k}}(\omega,T) = \frac{1}{2}\left[i\hat{\mathcal{G}}^{K}(\omega,T) - \hat{\mathsf{A}}(\omega,T)\right]$$

Approximations—

FSA $\hat{\Sigma}_{\mathbf{k}}(\omega,T) \to \hat{\Sigma}(\omega,T) \equiv \frac{1}{N\nu(0)}\sum_{\mathbf{k}} \hat{\Sigma}_{\mathbf{k}}(\omega,T)\,\delta(\xi_{\mathbf{k}})$

deep Migdal $E_F/\omega_0 \to \infty$

constant EDOS $\nu(\varepsilon) \to \nu(0)$

$$\hat{\Sigma}^{R}(\omega,T) = \frac{1}{2}\int_{-\infty}^{+\infty}\frac{\mathrm{d}\omega'}{2\pi}\int_{-\infty}^{+\infty}\frac{\mathrm{d}\nu}{\omega-\omega'-\nu+i0^{+}}\left\{iF^{K}(\nu,T)\,\check{\mathsf{A}}(\omega',T) + F^{\rho}(\nu,T)\,i\check{\mathcal{G}}^{K}(\omega',T)\right\},$$

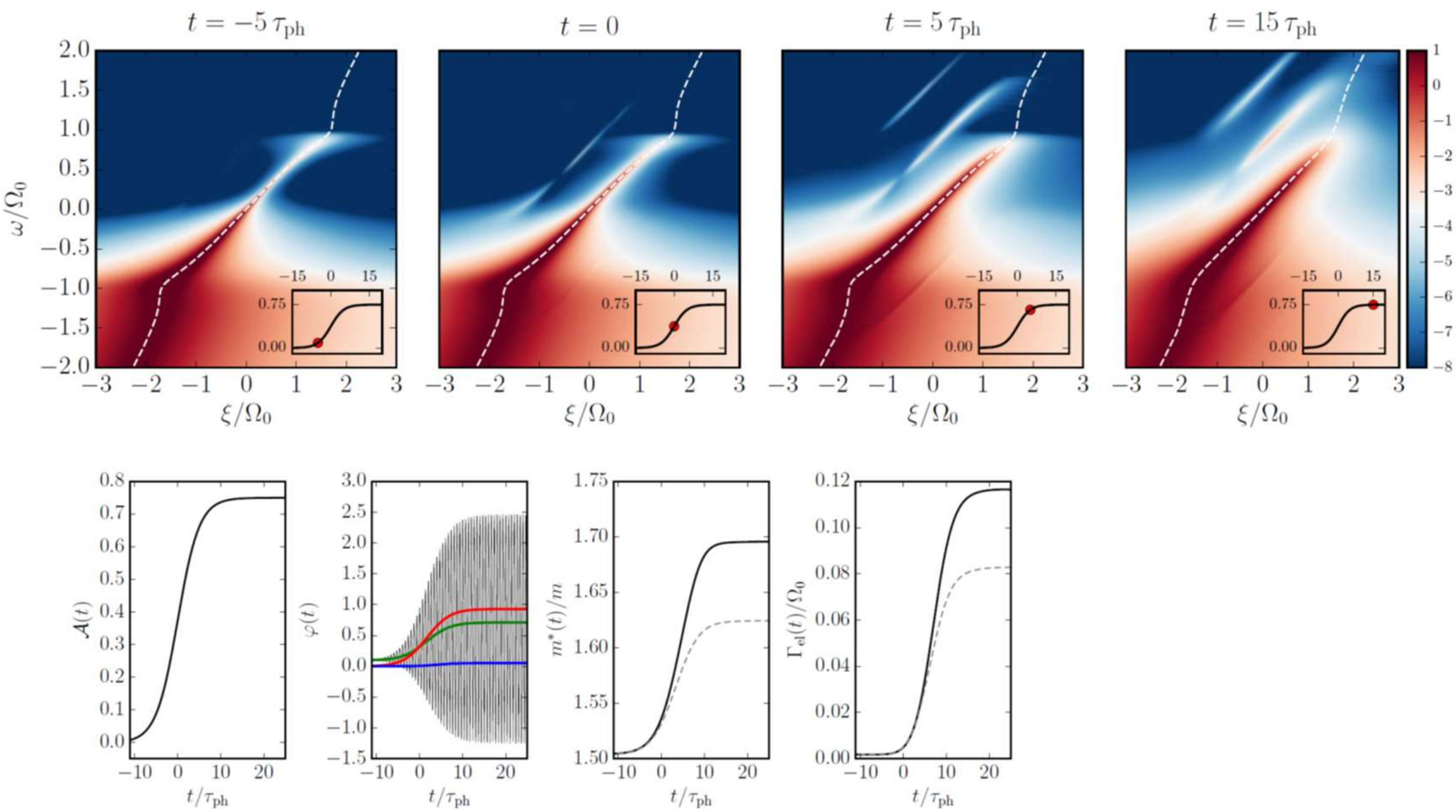
$$i\hat{\Sigma}^{K}(\omega,T) = \frac{1}{2}\int_{-\infty}^{+\infty}\frac{\mathrm{d}\omega'}{2\pi}\int_{-\infty}^{+\infty}\mathrm{d}\nu\,(2\pi)\delta(\omega-\omega'-\nu)\left\{iF^{K}(\nu,T)\,i\check{\mathcal{G}}^{K}(\omega',T) + F^{\rho}(\nu,T)\,\check{\mathsf{A}}(\omega',T)\right\}$$

spectral Eliashberg $F^{\rho}_{\xi,\xi'}(\nu,T) \equiv \frac{\nu(0)}{\nu(\xi)\,\nu(\xi')}\frac{1}{N^2}\sum_{\mathbf{k},\mathbf{k}'}|g_{\mathbf{k}\mathbf{k}'}|^2\,\frac{1}{2\pi}\,\rho_{\mathbf{k}-\mathbf{k}'}(\nu,T)\,\delta(\xi_{\mathbf{k}}-\xi)\,\delta(\xi_{\mathbf{k}'}-\xi'),$

Keldysh Eliashberg $iF^{K}_{\xi,\xi'}(\nu,T) \equiv \frac{\nu(0)}{\nu(\xi)\,\nu(\xi')}\frac{1}{N^2}\sum_{\mathbf{k},\mathbf{k}'}|g_{\mathbf{k}\mathbf{k}'}|^2\,\frac{1}{2\pi}\,iD^{K}_{\mathbf{k}-\mathbf{k}'}(\nu,T)\,\delta(\xi_{\mathbf{k}}-\xi)\,\delta(\xi_{\mathbf{k}'}-\xi')$

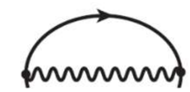
The real time Migdal-Eliashberg theory

Predictions for ARPES



Onset of pairing in non-equilibrium Floquet system

Introduce off-diagonal component of self-energy. Require self-consistency.



1. Calculate the Q-matrix—

Floquet cutoff (approx.)

$$(2\omega - m\Omega)\,\delta\mathcal{F}^R_{n,m} = -2\pi i\,\phi_{n,m} + \sum_{n'=-N_D}^{N_D}\left(\Sigma^R_{n',m-n+n'}\,\delta\mathcal{F}^R_{n-n',m+n'} + \Sigma^R_{n',m+n-n'}\,\delta\mathcal{F}^R_{n-n',m-n'}\right)$$

$$\delta\mathcal{F}^R_{n,m}(\omega) = \sum_{n',m'} \mathsf{Q}^{n,m}_{n',m'}(\omega)\,\phi_{n',m'}(\omega)$$

2. Solve the functional eigenvalue equation—

retarded interaction

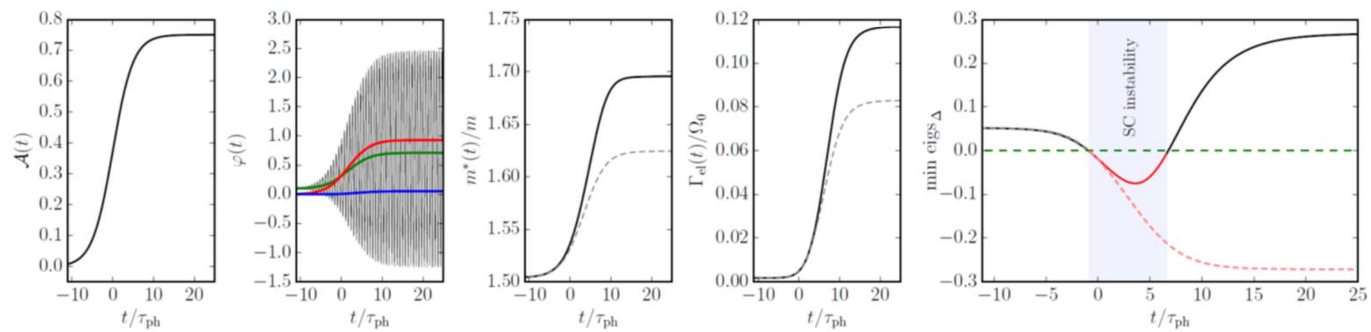
$$\Delta_n(\omega) = \frac{i\omega}{2\pi}\sum_{n'=-N_\phi}^{N_\phi}\sum_{n''=-N_D}^{N_D}\sum_{m'}\Bigg\{\mathsf{Q}^{n,0}_{n',m'}(\omega)\int_0^{+\infty}\frac{\mathrm{d}\omega'}{\omega'}K_{n''}(\omega - m'\Omega/2,\omega')\,\Delta_{n'-n''}(\omega')$$
$$-\left[\mathsf{Q}^{-n,0}_{n',m'}(\omega)\right]^*\int_0^{+\infty}\frac{\mathrm{d}\omega'}{\omega'}K^*_{n''}(\omega - m'\Omega/2,\omega')\,\Delta^*_{n'-n''}(\omega')\Bigg\}$$

dynamical self-energy effects (scattering, qp renormalization)

Compare to simple BCS

$$\Delta = -V\nu(0)\int d\xi\,\frac{\Delta}{|\xi|}$$

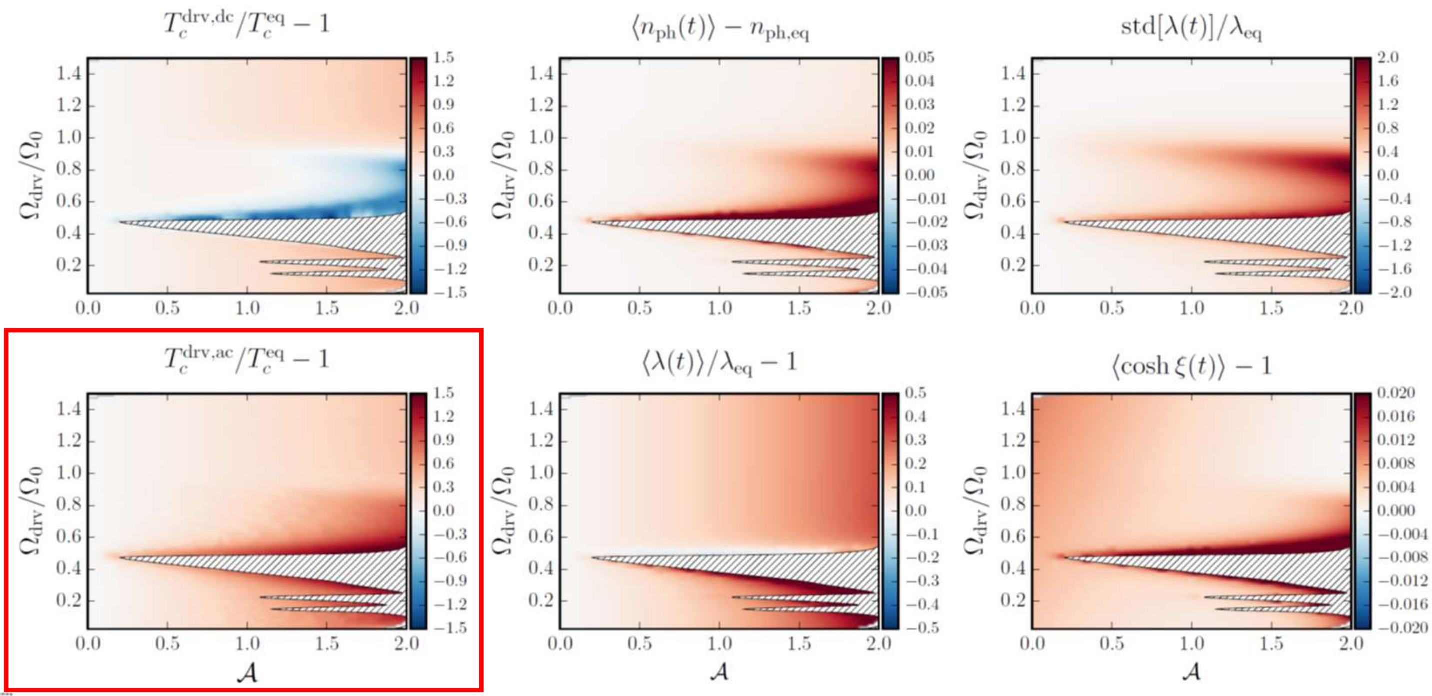
$$V = -\frac{g^2}{\omega_{\rm ph}}$$



Pairing instability in a driven electron-phonon system

Neglecting Floquet components. Use time averaged values

Include all Floquet components



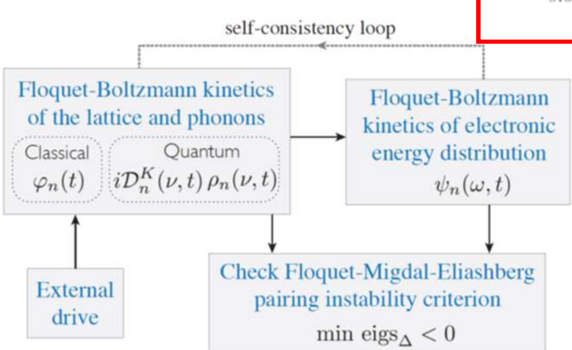


Photo-induced superconductivity
approach based on polaron transformation

M. Babadi, M. Knap, G. Refael, I. Martin, E. Demler

Phys. Rev. B 94, 214504 (2016)

Electron-phonon system in equilibrium: Lang-Firsov transformation

$$\hat{H}_{\text{el-ph}} = -J_0 \sum_{ij\sigma} c^\dagger_{i\sigma} c_{j\sigma} + \sum_k \omega_k b^\dagger_k b_k + \sum_{ik\sigma} \frac{\gamma_k e^{ikr_i}}{\sqrt{V}} (b_k + b^\dagger_{-k}) n_{i\sigma}$$

Apply unitary transformation $S = -\frac{1}{\sqrt{V}} \sum_{qj\sigma} \frac{\gamma_q}{\omega_q} e^{iqr_j} (b_q - b^\dagger_q) n_{j\sigma}$

$$e^S \mathcal{H}_{\text{el}-\text{ph}} e^{-S} = -\sum_{ij\sigma} J_{ij} c^\dagger_{i\sigma} c_{j\sigma} + \sum_{ij\sigma\sigma'} U_{ij} n_{i\sigma} n_{j\sigma'} + \hat{H}_{\text{ph}}$$

$$\boxed{J_{ij} = J_0 e^{-\frac{1}{\sqrt{V}} \sum_k \frac{\gamma_k}{\omega_k} (e^{ikr_i} - e^{ikr_j})(b_k - b^\dagger_{-k})}}$$

polaron dressing of electron tunneling

$$\boxed{U_{ij} = -\frac{1}{V} \sum_k e^{-ik(r_j - r_i)} \frac{\gamma_k^2}{\omega_k}}$$

phonon mediated attraction between electrons

Typical approach without drive: average over phonon equilbrium

Electron-phonon system out of equilibrium: LF transformation

$$\hat{H} = -\sum_{ij\sigma} J_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} + \sum_{ij\sigma\sigma'} U_{ij} n_{i\sigma} n_{j\sigma'} + \hat{H}_{\text{ph}}$$

$$J_{ij} = J_0 e^{-\frac{1}{\sqrt{V}} \sum_k \frac{\gamma_k}{\omega_k} (e^{ikr_i} - e^{ikr_j})(b_k - b^{\dagger}_{-k})}$$

polaron dressing of electron tunneling

$$U_{ij} = -\frac{1}{V} \sum_k e^{-ik(r_j - r_i)} \frac{\gamma_k^2}{\omega_k}$$

phonon mediated attraction between electrons

$$\hat{H}_{\text{ph}} = \hat{H}_{\text{ph},0} + \hat{H}_{\text{drv}}$$

$$\hat{H}_{\text{drv}} = A_k \omega_k^2 \cos \Omega t \; Q^{\text{R}}_k Q^{\text{R}}_{-k}$$

Simplest approach: average J_{ij} over a driven state of phonons

$$\langle J_{ij}(t) \rangle = J^0_{ij} + J^1_{ij} \cos 2\Omega t + \ldots$$

Floquet BCS type Hamiltonian

$$\mathcal{H}(t) = -J_{\mathrm{eq}} e^{-\zeta}(1 - A\cos 2\Omega t) \sum c_{i\sigma}^{\dagger} c_{j\sigma} + \sum U_{ij} n_i n_j$$

Move time dependence into interaction using the fact that modulation of the Hamiltonian as a whole has no effect

$$\tilde{H}(t) = J_{\mathrm{eq}} e^{-\zeta} \sum_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} - U(1 + \mathcal{A}\cos 2\Omega t) \sum_{i} n_{i\uparrow} n_{i\downarrow}$$

Solve Copper-pair instability problem

$$\frac{d}{dt}\langle c_{k\uparrow}^{\dagger} c_{-k\downarrow}^{\dagger}\rangle = 2i(\epsilon_k - \mu)\langle c_{k\uparrow}^{\dagger} c_{-k\downarrow}^{\dagger}\rangle - i\frac{U(1 + \mathcal{A}\cos 2\Omega t)}{V} \sum_{q} \langle c_{k\uparrow}^{\dagger} c_{-k\downarrow}^{\dagger}\rangle (n_{q\uparrow} + n_{q\downarrow})$$

$$- iU(1 + \mathcal{A}\cos 2\Omega t)(1 - 2n_k)\frac{1}{V} \sum_{q} \langle c_{q\uparrow}^{\dagger} c_{-q\downarrow}^{\dagger}\rangle$$

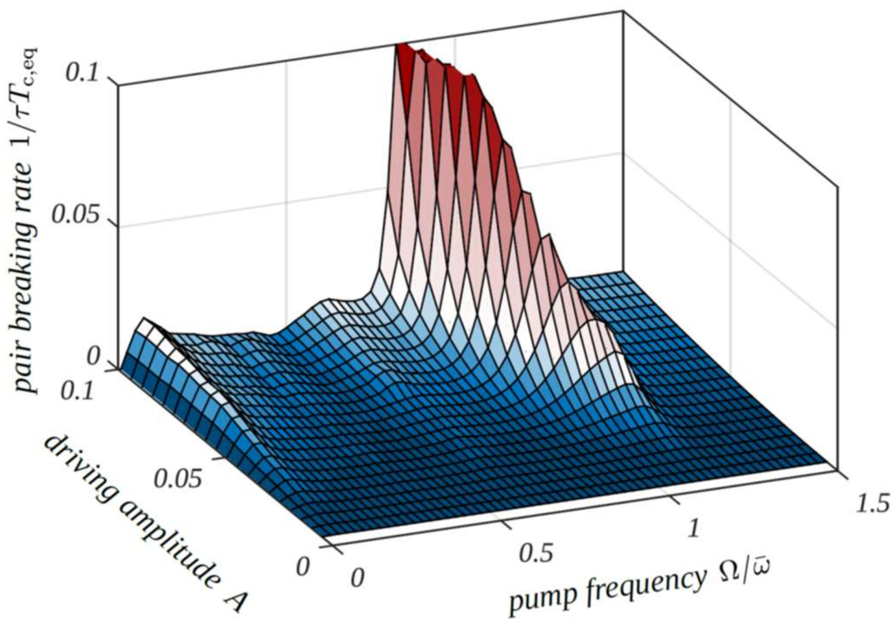
Appearance of complex eigenvalues signals instability

Need to include pair-breaking scattering of electrons due to non-equilibrium phonon state

Floquet Fermi's golden rule

Photoexcited phonons increase scattering of electrons which gives rise to pair-breaking

$$\frac{1}{\tau_{\mathrm{ph}}} = \frac{\pi}{2V} \sum_{qn} |\mathcal{F}_{qk_F}|^2 |\bar{\alpha}_{qn}|^2 \{(1 - n_{k_F - q}) \delta(2n\Omega - E_{k_F - q} - \omega)$$



Analysis of instabilities in Floquet BCS type Hamiltonian

Solve Copper instability problem with pair-breaking processes

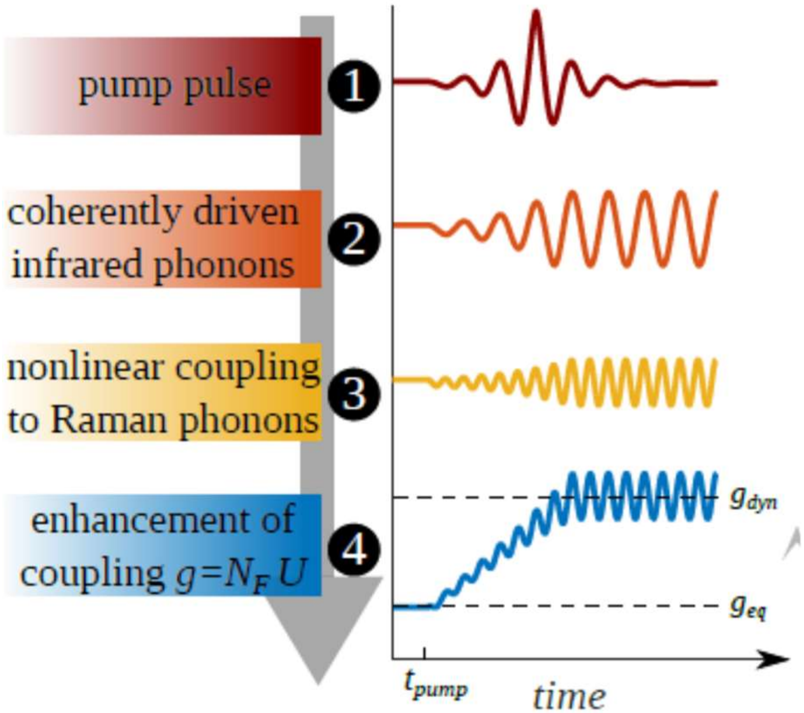
$$\frac{d}{dt}\langle c^\dagger_{k\uparrow}c^\dagger_{-k\downarrow}\rangle = 2i(\epsilon_k + \boxed{i/\tau} - \mu)\langle c^\dagger_{k\uparrow}c^\dagger_{-k\downarrow}\rangle - i\frac{U(1+\mathcal{A}\cos 2\Omega t)}{V}\sum_q \langle c^\dagger_{k\uparrow}c^\dagger_{-k\downarrow}\rangle(n_{q\uparrow}+n_{q\downarrow}) - iU(1+\mathcal{A}\cos 2\Omega t)(1-2n_k)\frac{1}{V}\sum_q \langle c^\dagger_{q\uparrow}c^\dagger_{-q\downarrow}\rangle$$

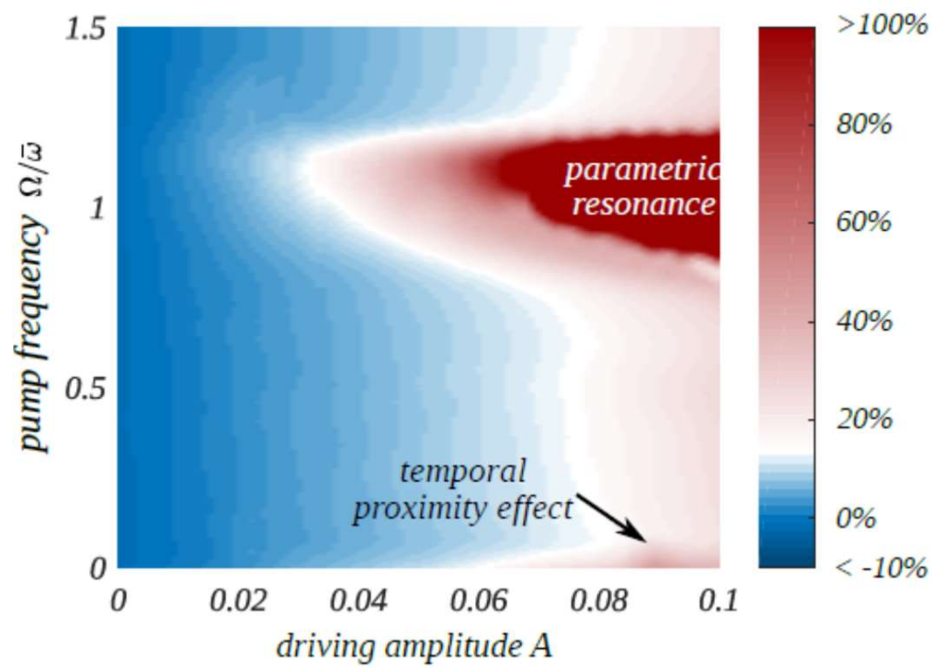
Appearance of complex eigenvalues signals instability

Pairing instability in a driven electron-phonon system

$$\hat{H}_{\text{el-ph}} = -J_0 \sum_{ij\sigma} c^{\dagger}_{i\sigma} c_{j\sigma} + \sum_k \omega_k b^{\dagger}_k b_k + \sum_{ik\sigma} \frac{\gamma_k e^{ikr_i}}{\sqrt{V}} (b_k + b^{\dagger}_{-k}) n_{i\sigma}$$

$$\hat{H}_{\text{drv}} = A_k \omega_k^2 \cos \Omega t \;\; Q^{\mathrm{R}}_k Q^{\mathrm{R}}_{-k}$$





Conclusions

External drive leads to enhancement of electron-phonon interaction. It can be understood as parametric amplification or result of a squeezed state of phonons. This leads to an increase in the effective BCS coupling constant

This also results in additional scattering of electrons that leads to pair-breaking

We find that increase in BCS coupling can dominate and find possible increase of instability temperature by 150%. Floquet aspects are crucial.