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NONLINEAR DYNAMICS IN MULTIMODE OPTICAL FIBERS: RECENT ADVANCES

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Nonlinear optics in multimode fibers (MMFs) has had a renaissance over the past two decades, driven by both basic and applied research. MMFs provide an ideal setting for studying multidimensional systems with their complicated collective dynamics. The uniqueness of MMF including the spatial degree of freedom, spatiotemporal dynamics and inherent disorder make them an ideal tool for exploring novel physics beyond communication. Here, we briefly discuss an overview of nonlinear dynamics in MMFs by focusing their applications in spatiotemporal pulse shaping, self-beam cleaning, and broadband continuum generation. The nonlinearities in MMFs can be useful in linear and nonlinear imaging in microscopy and endoscopy configurations. The growing interest among researchers for nonlinearity in MMFs fibers is pretty evident, indicating a growth in the value of MMFs.

Keywords: multimode fibers, graded-index fiber, nonlinear dynamics, spatiotemporal pulse shaping, Kerr self-beam cleaning, broadband continuum generation.

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1. Introduction. In the 1970s, the MMFs were initially employed for nonlinear optics other than their usages in optical communication because most optical fibers available at that time allowed several modes [1, 2]. The scenario changed in the 1980s, when single-mode fibers were marketed for use in telecommunication applications. After 2005, there was a renewed interest in MMFs, owing in part to their potential for space-division multiplexing in optical communication systems [3–5]. Furthermore, the transmission of many modes in the same fiber is expected to enable a slew of unique applications and approaches that extend beyond signal processing and communications. As a result of this interest, nonlinear effects in MMFs have been widely explored since 2010 [6-8]. Since each mode in a fiber has its own spatial distribution and propagation constants with a particular dispersion characteristic, it is possible to fine-tune the characteristics of the modes by carefully constructing the cross-sections of the fiber. Also, the interaction of multiple modes can add numerous degrees of freedom in attaining interesting nonlinear dynamics, allowing the implementation of efficient processes over previously unreleased enormous bandwidth values. A potent method for effectively producing, for instance, optical sources in the MIR based on parametric interactions or ultra-broadband supercontinuum radiation, is intermodal nonlinear interaction [9]. Simultaneously, controlled multimode propagation may be employed over short distances for high resolution image transmission or can open the door to new options in optical manipulation of multiple particles via spatially controlled and programmable beam patterns.

Since the introduction of space-division-multiplexing (SDM) in the early 2000s, MMFs have received fresh attention, fueled by innovative exact production processes as well as revolutionary devices that enable mode control with remarkable modal purity [9, 10]. Aside from their traditional use for short-distance communication links, MMFs have received mentionable recent

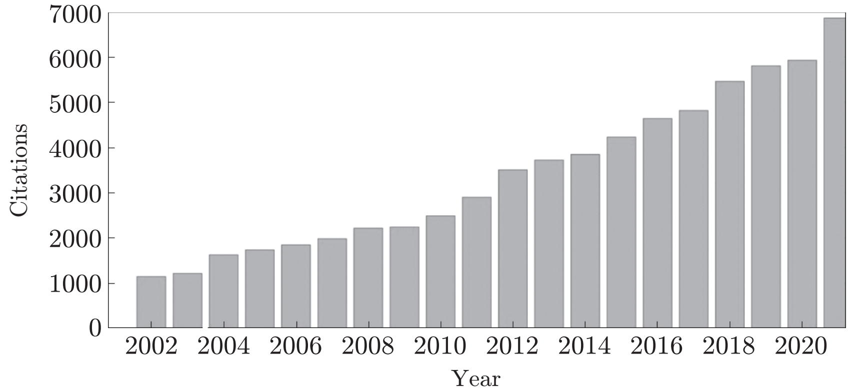

Fig. 1. Number of citations results for last two decades when the keyword "Nonlinearity in Multimode Fibers" was googled at Google Scholar platform on November 4, 2022

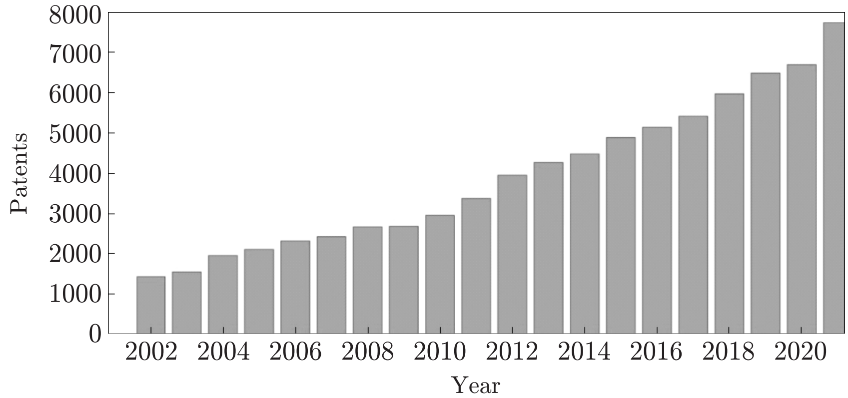

Fig. 2. Number of patents in last twenty years for the keyword "Nonlinearity in Multimode Fibers" at Google Scholar platform on November 4, 2022

research attention for coherent image transmission [11, 12], mode-division multiplexing for high-speed optical communications [13, 14], quantum research [15], and multimode complex nonlinear optics [16, 17]. Although SDM is established on the linear region propagation of individual information channels through discrete (non-interacting) spatial modes, interest in nonlinear events involving the interaction of several spatial modes has grown during the past ten years. Indeed, the interaction of several modes results in freshly untapped, complex nonlinear dynamics that increase the possibilities of the single mode platform. In a simplified situation, intermodal nonlinear dynamics may be seen as the outcome of interactions between numerous unique pairings of modes. Despite not being thorough or fully reflecting the complexity [10], this method still provides a straightforward understanding and design guidelines about broadband operation. The latter is the most adaptable of the nonlinear processes in silica fibers, which include stimulated Raman scattering (SRS), stimulated Brillouin scattering (SBS), and Kerr-driven four-wave-mixing (FWM).

The nonlinearity in MMFs is attracting significant attention of researchers, as seen in Fig.1 and Fig. 2. Here, we made histograms with the number of search results for citation and patents for last twenty years in the Google scholar platform. The nonlinearity in MMFs is being utilized in various research directions. Here, we briefly covered few of them in the following fashion: Section 2 brushes up some basics, characterstics, and types of MMFs, section 3 explains the emerging spatiotemporal pulse shaping in MMFs, section 4 discusses the self-beam cleaning

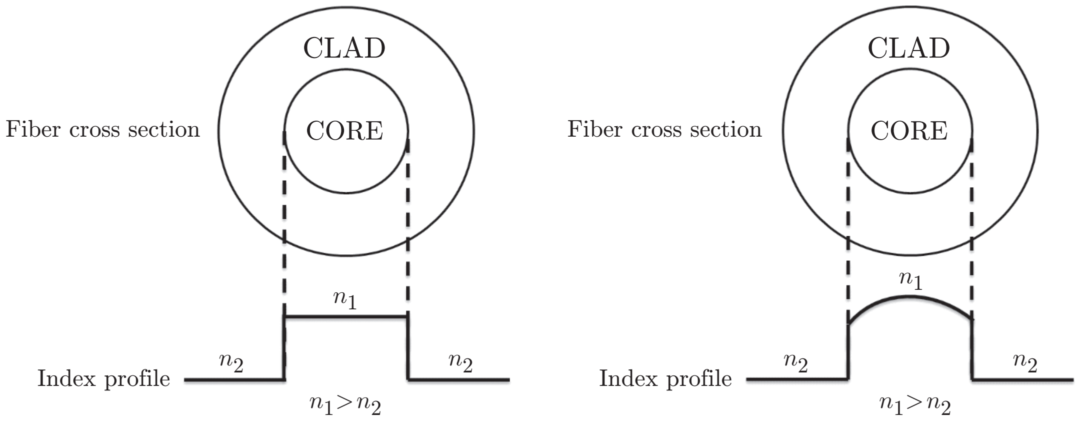

Fig. 3. Schematic of the refractive index profiles of step-index and graded-index optical fibers

technique in the MMFs, section 5 present the broadbad continuum geneartion in MMFs and some of our new experimental results, and section 6 concludes our paperalong with the new possiblities of nonlinearity in MMFs.

2. Multimode fibers. In general, the fibers which support more than one mode at a given wavelength are either referred as few mode fibers or known as MMFs. MMFs generally have a large core diameter ranging between 50-100 μm which may increase the transmission capacity in terms of spatial division multiplexing and enables for higher power transmission due to large mode area. As light waves pass through the core, they are dispersed into various routes, forming various mode patterns, and these different paths result into the distortion at the receiving end. Due to its employment in local-area networks, multimode cables are regarded as the "domestic" kind of fiber.

Typically, graded index (GRIN) fiber and solid core step-index (SI) fiber are the two variants of MMFs. In SI fiber, the core's refractive index is uniform along its diameter and rapidly drops at its margins (cladding). In contrast, GRIN fiber contains a core whose refractive index varies along its diameter. SI fibers allow multimode transmission; however distortion occurs owing to intermodal dispersion, whereas in case GRIN fibers, intermodal dispersion minimizes due to self-focusing effect owing to nearly parabolic refractive index distribution in the core region. The schematics of the refractive index profiles of SI and GRIN fibers are shown in Fig. 3.

A variety of applications also make use of specialty MMFs such W-type fiber [18, 19], double-clad fiber [20], triple-clad fiber [21, 22], multimode photonic crystal fiber (MM-PCF) [23, 24], multilayered fibers, and multi core fibers. If used below the fiber's cutoff wavelength, even single-mode fibers can accommodate multiple modes.

Crosignani et al. presented a set of coupled-mode nonlinear Schrödinger equations (NLSEs) in 1982 to explain the pulse propagation in MMFs with an intensity-dependent refractive index [25]. In 2008, Poletti and Horak reported an expanded form of the multimode NLSEs (or MMGNLSEs) for MMFs by incorporating wavelength-dependent mode coupling, nonlinear coefficients, high-order dispersion, Kerr and Raman nonlinearities, and self-steepening effects [26]. After that, Mafi gave a thorough examination of the modal characteristics, dispersive behavior, and nonlinear mode coupling in GRIN MMFs in 2012 [27].Pedersen et al. later developed an enhanced version of the MM-GNLSEs that accounts for the dispersion of the transverse field distributions [28]. In 2013, Khakimov et al. proposed a numerical approach for solving the MM-GNLSEs [29]. Recently, Wright et al. proposed a parallel numerical solution approach for the MM-GNLSE system which is available to download for free [30]. The well-known split-step Fourier approach may be used to solve the MMGNLSE in the time domain or in the frequency domain while the solution carried out in the frequency domain is more effective and takes less time [31, 32].

3. Spatiotemporal pulse shaping. Since long, nonlinearities like modal-phase matching of four-wave mixing (FWM) processes and other nonlinear optical phenomena are very well-known in MMFs. The ability to manipulate the temporal and spectral characteristics of ultra-short pulses while utilizing the degrees of freedom offered by fiber multimodality, however, is a research area that has only recently begun to take off. Speckle-like events in space, frequency, and time occur during the propagation through MMFs and the superposition of multiple guided modes and modal dispersion cause distortions. For a variety of applications, it is desirable to eliminate and make advantage of the speckle-like distortions brought on by modal dispersion scattering in MMFs. Spatial focusing via MMF has been performed utilizing wave front shaping controlled by diverse approaches such as transmission matrix measurements [33, 34] and optical phase conjugation [35], which was inspired by wave front-shaping research in bulk scattering materials. Rokitski et al. used time-gated spatial heterodyne interferometry to characterize the propagation of an ultra-short pulse in a multimode optical fiber [36], whereas Guang et al. used a technique for the complete measurement of the output pulses called a spatially and temporally resolved intensity and phase evaluation device: full information from a single hologram [37]. Morales-Delgado et al. described a wavefront-shaping approach for delivering ultra-short pulses across MMF, in which short pulses transmitted through and distorted by the MMF interfere with an ultra-short reference pulse to generate a time-gated digital hologram including spatial phase information [38, 39]. The spatially shaped reference counter travelling via the same MMF with spatial mode-selective phase conjugation mechanism supplied the brief pulses. Such research holds promise for nonlinear optical imaging through fiber and may have applications in optical communications.

In addition, ultra-short pulse propagation with complex modal interactions in graded-index multimode fibers (GIMFs) revealed novel nonlinear dynamics for beam shaping, frequency conversion, and ultra-short pulse generation. The production of high-quality beam forms in spatiotemporal mode-locked fiber lasers, complicated multimode laser structures, is reported by modifying the temporal dynamics of mode. With spatiotemporal mode-locking, -locked pulses and beam profile enhancement is accomplished. For an optical pulse travelling across an MMF, long-range spatio-temporal intensity correlations have been investigated [40]. After an MMF, it has been demonstrated that the averaged light intensity may be controlled temporally at the expense of the spatial pattern [41]. A single MMF probe has recently been used to show a number of nonlinear optical imaging methods, including coherent anti-Stokes Raman scattering (CARS) microscopy [42], two-photon excitation microscopy [43, 44], and 3D micro fabrication based on two-photon polymerization [45]. All these nonlinear imaging techniques need control over several spatial modes on the MMF input via spatial-domain wave front shaping. Recently, Guo et al. have demonstrated the realization of wavelength-tunable spatiotemporal mode locking in a partial MMF laser with a linear cavity [46]. In fact, spatial light modulators, which are fundamentally free-space components, are used in wave front-shaping techniques. This research may open up new possibilities for MMF-based space-division multiplexed optical communications or nonlinear microscopy and imaging.

4. Self-beam cleaning. Since the last two decades, researchers have been studying the self-beam cleaning (SBC) effect in MMFs. The technique is based on spatial beam self-cleaning by the nonlinear Kerr effect within the MMF, which is accomplished by modulating the spatiotemporal pulse and can be controlled using suitable injection conditions [47]. The SBC process causes the spatial beam profile to evolve owing to intermodal four-wave mixing, cross-phase modulation, and group delay dispersion effects. Multiple modes interact, resulting in periodic longitudinal intensity pattern modulation, which creates long-periodic gratings by

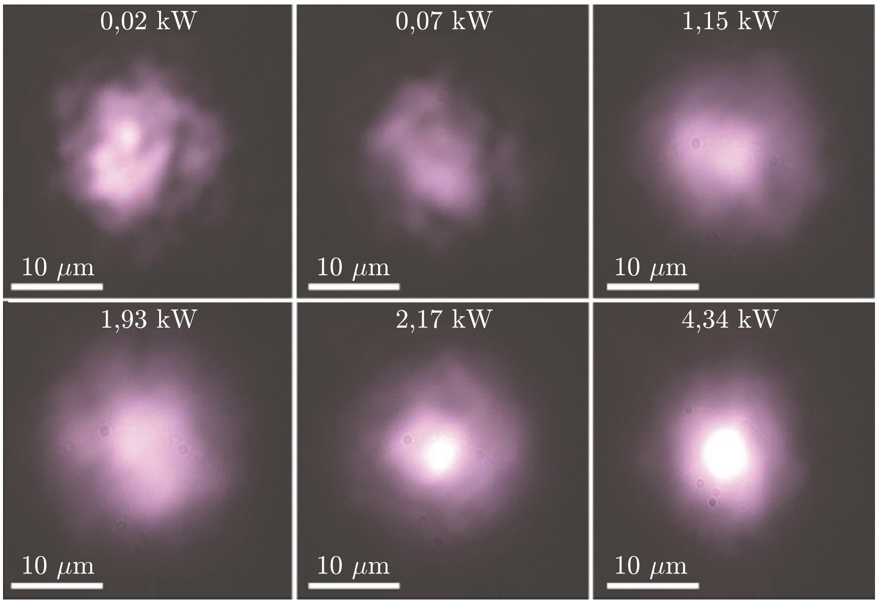

Fig. 4. Near-field spatial distributions for normal incidence with increasing input peak pump powers recorded at the output of GRIN MMF leading to LP_{01} mode. Adapted with permission from Ref. [53]

utilizing the nonlinear Kerr effect. This facilitates energy transfer across the transverse modes and for phase-matching processes to occur [48–50]. The process is non-reciprocal because of self-phase modulation and energy exchange among the guided modes. As a result, the energy sent into the lower-order transverse mode is irreversibly locked in that mode [51]. Experimental investigations in MMFs reveal a nonlinear phenomenon named Kerr-induced beam self-cleaning (KBSC). This nonlinear process causes reshaping random speckle pattern in MMFs into self-cleaned transverse modal profile at higher power levels and is shown in Fig.4. Investigations are still being conducted for in-depth theoretical study addressing the intricate mechanism of SBC. However, the process of SBC nonlinear reshaping can be accurately simulated numerically [17]. Different fiber types, including GRIN MMFs [16, 47, 52–55], step-index MMFs [56, 57], microstructure MMFs [57, 58], and tapered fiber [58, 59], as well as different pump lasers with pulse width varying from femto-second to nanosecond duration [47, 52] and normal to anomalous dispersion regimes [57] have all been used in experimental setups to demonstrate the SBC and other similar processes. It should be highlighted that, over long distances, the careful integration of beam-cleaning techniques with dissipation gives rise to intriguing possibilities for designing high-performance light sources.

5. Broadband continuum generation. Broadband continuum production is the result of the interaction of many nonlinear processes in which a narrow-band pulse experiences significant spectral widening [60], visualized in Fig. 5 [61]. Alfano and Shapiro discovered it in 1970 [62]. It is used in a variety of fields, including high-precision frequency metrology [63], optical coherence tomography [64], molecular spectroscopy [65], and so on. Continuum creation in MMF fiber extends the capabilities of the now-common single-mode fiber continuum. With phenomena such as intermodal four wave mixing processes and cascading Raman production that may be controlled by fiber length [66], multimode continuum generation is complicated, as shown in Figs. 6 and 7. Nevertheless, continua formed in MMF may be somewhat regulated [67, 68], and appropriate beam quality can be attained across a large spectrum range for some applications [65, 69, 70]. Scaling to increased pulse energy and power is caused by larger mode areas and the existence of modal dispersion in MMFs [63, 71].

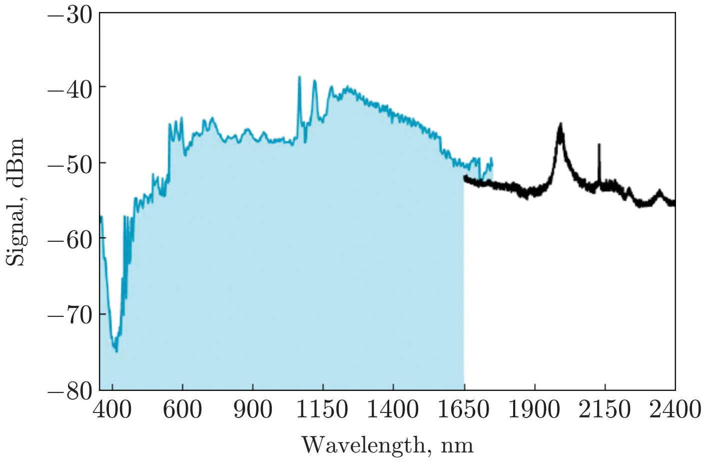

Fig. 5. Experimental supercontinuum spectra obtained in a 28.5 m long graded index MMF using 185 kW peak pump power at 1064 nm. Spectrum recorded using two different OSAs covering the spectral range from 350 to 2400 nm. Adapted with permission from Ref. [59]

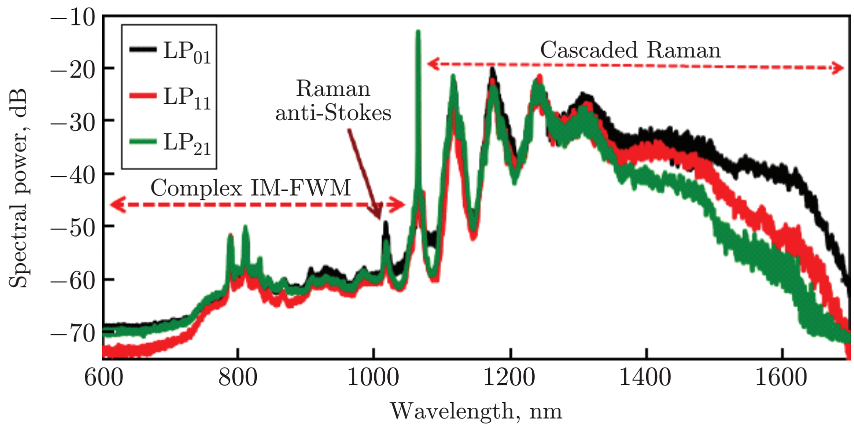

Fig. 6. Comparative output spectra for three different spatial modes, LP_{01}, LP_{11} and LP_{21} at fixed input pump peak power of 5.25 kW. Adapted with permission from Ref. [53]

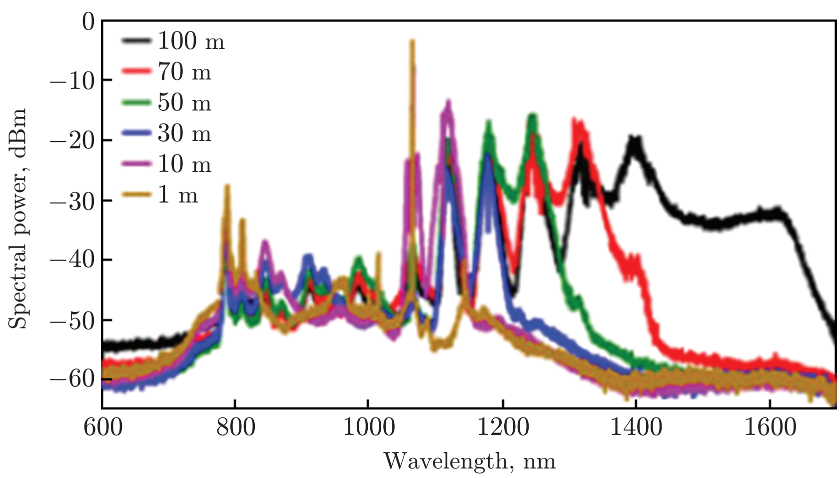

Fig. 7. Spectral evolution with different fiber lengths at fixed average constant input pump power of 35 mW. Adapted with permission from Ref. [66]

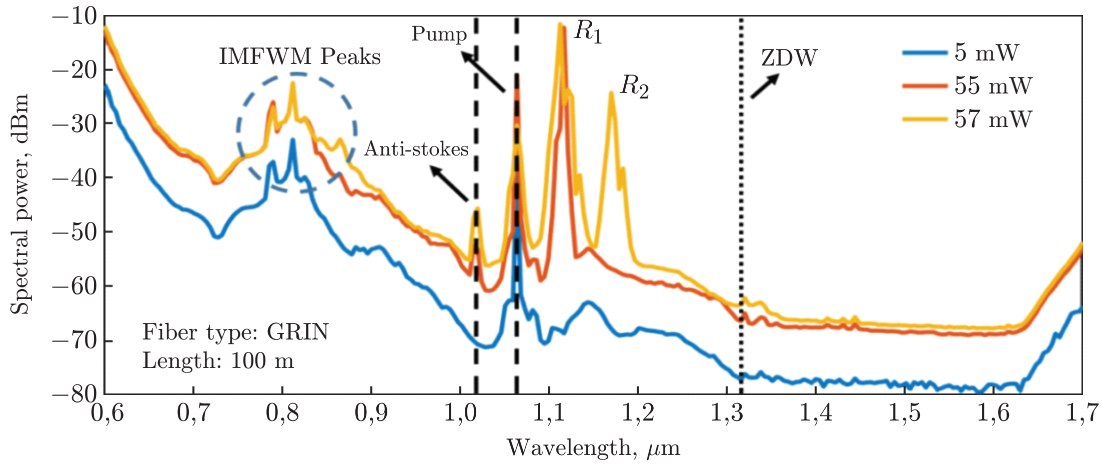

Fig. 8. The spectral power with respect to the wavelength at different coupled average powers in 100 m long GRIN-MMF by pumping at 1064 nm wavelength. R_1: First-order Raman Stokes; R_2: Second-order Raman Stokes; ZDW: Zero dispersion wavelength

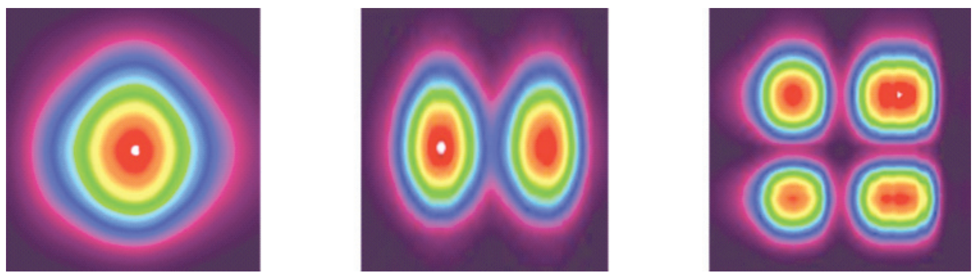

Fig. 9. Selective excitation of spatial modes (LP_{01}, LP_{11}, and LP_{21}, respectively) in the GIF at 1064 nm

We experimentally observed the intermodal four wave mixing (IMFWM) and cascaded Raman scattering (CRS) in MMFs at 1064 nm pump wavelength, which lies in the normal dispersion region of the fibers under test, namely the GRIN and the non-zero Dispersion Shifted Fiber (NZDSF). The NZDSF is a single-mode fiber, which shows multimode behavior at 1064 nm. The spectral evolution was observed as light was launched into a 100 m long piece of standard GIF (Thorlabs GIF625-100) with a core diameter 62.5 μm and NA 0.275. The output spectrum of the fiber at different coupled average powers are shown in Fig. 8.

Here, we can clearly see the generation of sidebands on both the sides of the pump (stoke and anti-stoke lines) which increases on increasing the coupled power into the fiber, which is due to CRS [66] where, under suitable pump power conditions, the first Raman Stokes line extracts enough power from the pump to seed the generation of the next Stokes lines, which produces several Raman peaks via a cascaded process. Peaks around 0.8 μm are governed by IMFWM, due to the absence of soliton effect, as the normal dispersion region inhibits soliton operation. The presence of IMFWM process hints the multimodal characteristic of the fiber. To further confirm the presence of multiple modes, we selectively excited the LP_{01}, LP_{11} and LP_{21} modes, as shown in Fig. 9 by manipulating the fiber using the six-axis stage at the pump wavelength.

In a similar way, the pump radiation is coupled in 100 m long NZDSF and the spectrum is recorded in optical spectrum analyzer, as shown in Fig.10. The Raman peaks become prominent

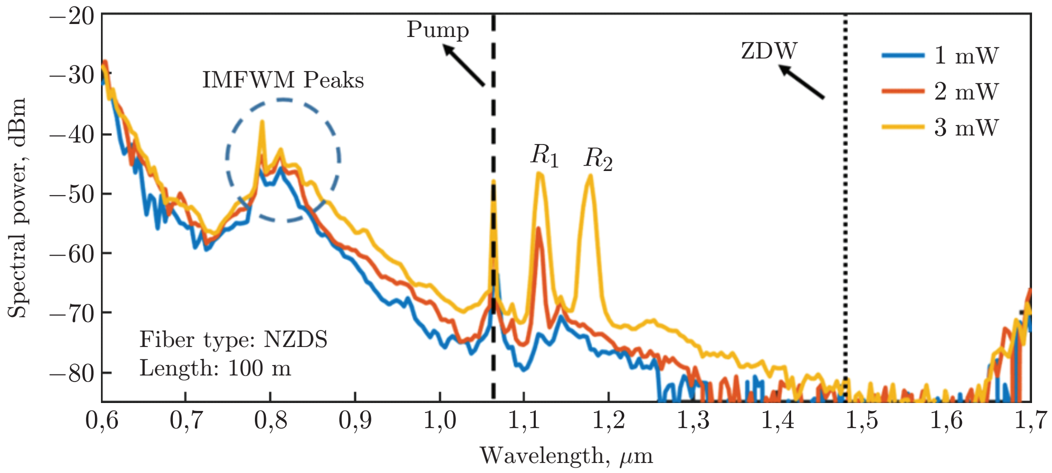

Fig. 10. Spectral power variation for different coupled average powers in 100 m long NZDS fiber, pumped at wavelength of 1064 nm

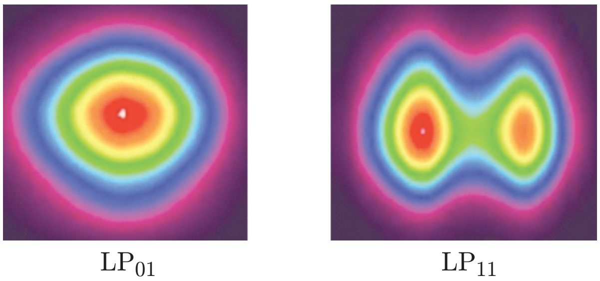

Fig. 11. Experimental Selective excitation of spatial modes (LP_{01} and LP_{11} respectively) in the NZDSF at 1064 nm

on increasing the power, while around 0.8 μm wavelength, we observe IMFWM [66] because the fiber acts as multimode at the pump wavelength, as mentioned earlier.

The LP_{01}, and LP_{11} modes were excited in the fiber as shown in Fig. 11, which further confirms the multimode characteristics of the fiber at the given wavelength. The NZDSF owns a smaller effective mode area, compared to that of the GIF; this triggers nonlinearity in the NZDSF prominently as compared to that of the GIF, thus generating more peaks at lower coupled power.

6. Conclusions. To summarize, it is evident that the MMF acts a major contributor to the field of nonlinear optics with significant contributions, which include broadband continuum generation, beam cleaning and spatiotemporal pulse shaping and etc., along with a number of advantages like larger core diameter, which provides shorter termination time, cheaper connectors and components as compared to single mode fibers and compatibility with high-speed data transmission. Therefore, it might be possible to realize several advanced photonic systems with enhanced or new capabilities through MMFs. However, understanding and design of highly multimode nonlinear optical systems certainly present major challenges, as multi-mode cables are thicker and their prices are twice as expensive as single-mode cables. But, advances in understanding of multimode systems, along with development of optical components and enhanced computing resources, create a strong foundation for addressing these challenges. Armed with the tools of modern and advanced technology, we expect that future photonics engineers will design nonlinear multimode devices and instruments in ways that are presently challenging or impossible and may have major impact on optical sciences.

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